

Bose-Einstein condensation of photons in the matter-dominated universe

Ze Cheng

School of Physics, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

zcheng@mail.hust.edu.cn

ABSTRACT

In 1914, Planck introduced the concept of a white body. In nature, no true white bodies are known. We assume that the universe after last-scattering is an ideal white body that contains a tremendously large number of thermal photons and is at an extremely high temperature. Bose-Einstein condensation of photons in an ideal white body is investigated within the framework of quantum statistical mechanism. The computation shows that the transition temperature T_c is a monotonically increasing function of the number density n of photons. At finite temperature, we find that the condensate fraction $N_0(T)/N$ decreases continuously from unity to zero as the temperature increases from zero to the transition temperature T_c . Further, we study the radiation properties of an ideal white body. It is found that in the condensation region of $T < T_c$, the spectral intensity $I(\omega, T)$ of white body radiation is identical with Planck's law for blackbody radiation.

Subject headings: cosmology: cosmic background radiation — cosmology: theory — cosmology: inflation

1. Introduction

Nowadays it is recognized that the Bose-Einstein condensation (BEC) is a common quantum property of the many-particle systems in which the number of particles is conserved. If particles are bosons, the interaction between particles must be repulsive, whereas if particles are fermions, the interaction must be attractive. The generalized BEC is the macroscopic accumulation of Bose particles in the energetic ground state below a critical temperature. In 1995, the three research groups in the U. S. A. observed the BEC of ultracold Bose atomic gases in a trapping potential (Anderson 1995; Davis 1995; Bradley 1995). In 1938, Kapitza discovered the superfluid phase of liquid ^4He below 2.17 K, which can be viewed as a BEC among strongly interacting ^4He atoms. In 1911, Onnes discovered the superconducting state of metal mercury below 4 K. In 1957, Bardeen, Cooper, and Schrieffer provided a successful microscopic description of superconductors in terms

of Cooper pairs (Cooper 1956; Bardeen 1957). The superconducting state can be regarded as a manifestation of the BEC of Cooper pairs. The BEC has been observed also in several systems of solid-state quasiparticles, which include excitons (Butov 2002; Eisenstein 2004), exciton-polaritons (Deng 2010), and magnons (Nikuni 2000; Demokritov 2006).

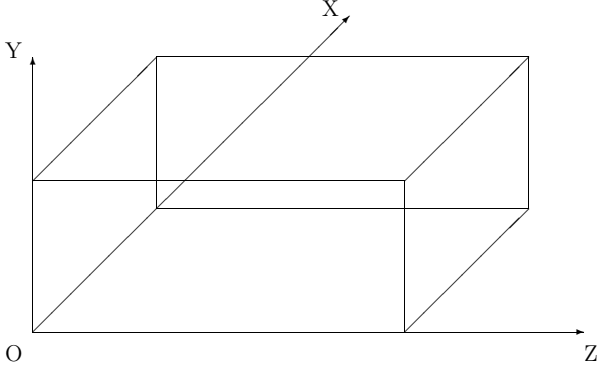
However, the most omnipresent blackbody radiation does not show this phase transition. The photons in blackbody radiation have a vanishing chemical potential, so that the number of photons is not conserved when the temperature of the blackbody is varied; at low temperatures, photons are absorbed by the walls of the blackbody instead of occupying the zero-momentum state. Theoretical works have considered thermalization processes that conserve photon number, involving Compton scattering with a gas of thermal electrons (Zel'dovich 1969) or photon-photon scattering in a nonlinear resonator configuration (Chiao 1999, 2000; Bolda 2001). Number-

conserving thermalization was experimentally observed (Klaers 2010a) for a two-dimensional photon gas in a dye-filled optical microcavity, which acts as a ‘white-wall’ box. In the presence of thermalization processes that conserve photon number, Weitz and colleagues have observed the BEC of two-dimensional photons in a dye-filled optical microcavity (Klaers 2010b, 2012). Kirton and Keeling have established a nonequilibrium model of photonic BEC in the dye-filled microcavity (Kirton 2013).

Now, let us turn to a gas of three-dimensional photons in the universe. The evolution of the universe according to the standard hot big-bang model (Turner 1999) is sketched as follows: (1) Radiation-dominated phase. At the age of the universe earlier than about 10000 years, when the temperature of the universe exceeded $k_B T \gtrsim 3$ eV, the energy density in radiation and relativistic particles exceeded that in matter. At the earliest times, the energy in the universe consists of radiation and seas of relativistic particle-antiparticle pairs. At the time of 10^{-11} second, when $k_B T \sim 300$ GeV, the sea of relativistic particles includes six species of quarks and antiquarks, six types of leptons and antileptons, and twelve gauge bosons. When $k_B T \lesssim 2mc^2$ where m is the mass of a particle species, those particles and their antiparticles annihilate into at least two photons. When the universe was seconds old and the temperature was around 1 MeV, big-bang nucleosynthesis led to the production of the light elements D, ^3He , ^4He , and ^7Li . When $k_B T \ll 1$ MeV, the last of the particle-antiparticle pairs, the electrons and positrons, annihilated. (2) Matter-dominated phase. When the temperature reached around $k_B T \sim 3$ eV, at a time of around 10000 years the energy density in matter began to exceed that in radiation. Shortly after matter domination begins, photons in the universe undergo their last-scattering off free electrons. Last scattering is precipitated by the recombination of electrons and ions (mainly free protons), which occurs at a temperature of $k_B T \sim 0.3$ eV because neutral atoms are energetically favored. Before last-scattering, matter and radiation are tightly coupled; after last-scattering, matter and radiation are essentially decoupled. Under the surface of last scattering, the number of photons in the universe is conserved.

In 1914, Planck introduced the concept of a white body (Planck 1914). A white body is one for which all incident radiation is reflected uniformly in all directions, an idealization exactly opposite to that of the blackbody. In nature, no true white bodies are known. We assume that the universe after last-scattering is an ideal white body that contains a tremendously large number of thermal photons and is at an extremely high temperature. As shown in Fig. 1, an ideal white body can be regarded as a rectangular cavity whose walls are high-reflecting planar optical mirrors and are kept at a constant temperature T . This white body contains only thermal radiation in its interior. Because the cavity walls absorb no electromagnetic radiation, the number of photons is conserved in the white body. In the present paper, we shall study the BEC of photons in an ideal white body. The BEC of photons in an ideal white body takes place in the momentum space. The planar mirrors provide a chemical potential for a photon, making the system formally equivalent to a three-dimensional gas of number-conserving, massless bosons. The advantages of an ideal white body are as follows: (1) the loss rate is zero and (2) the nonlinearity does not exist.

The important properties of such a BEC will be expounded in the following. At first, we investigate the BEC of noninteracting photons in an ideal white body. It is found that such a BEC is a second phase transition. The expression of transition temperature T_c is obtained, which is dependent on the number density n of photons. At finite temperature, we find that the condensate fraction $N_0(T)/N$ decreases continuously from one to zero as the temperature increases from zero to the transition temperature T_c . We find that the vacuum we currently observe is the condensate or superfluid of photons. Further, we study the radiation properties of an ideal white body. It is found that in the condensation region of $T < T_c$, the spectral intensity $I(\omega, T)$ of white body radiation is identical with Planck’s law for blackbody radiation. Our investigation into the BEC of three-dimensional photons deepens our understanding of the development of the universe as an ideal white body. Since the BEC of three-dimensional photons was never explored previously, we point out here that there are new features, which we believe are worthy of exploration. The predicted properties of the



1

Fig. 1.— An ideal white body: a rectangular cavity enclosed by perfectly conducting walls and kept at a constant temperature.

BEC of photons in an ideal white body are highly relevant to the present universe.

2. Formation of photonic Bose-Einstein condensates

Because the number of photons is conserved in a white body, the photon system possesses a non-vanishing chemical potential μ . Now we need to quantize the electromagnetic field. In terms of the creation and annihilation operators $a_{\mathbf{k}\sigma}^\dagger$ and $a_{\mathbf{k}\sigma}$, respectively, of circularly polarized photons with wave vector \mathbf{k} and helicity $\sigma = \pm 1$, the grand canonical Hamiltonian \mathcal{H}_{em} of the photon system is given by

$$\mathcal{H}_{em} = \sum_{\mathbf{k}\sigma} (\hbar\omega_{\mathbf{k}} - \mu) a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma}. \quad (1)$$

where the zero-point energy terms are dropped. Here \hbar is Planck's constant reduced, $\omega_{\mathbf{k}} = c|\mathbf{k}|$ is the angular frequency of a photon, and c is the propagation velocity of light in the vacuum. Equation (1) represents the grand canonical Hamiltonian of the system of noninteracting photons.

In the representation with a set of quantum numbers $i \equiv \{\mathbf{k}, \sigma\}$, an energy level of a three-dimensional photon is denoted by the quantum number i . In the grand canonical ensemble, the system under study consists of N noninteracting photons, which are distributed over various quantum states i and have a chemical potential μ . Based on the first principles of statistical mechanics, one knows that the average number $\langle N_i \rangle$ of photons in the i th state of energy E_i obeys the Bose-Einstein distribution

$$\langle N_i \rangle = \frac{1}{e^{\beta(E_i - \mu)} - 1}, \quad (2)$$

where β is related to the temperature T by $\beta = 1/k_B T$ and k_B is the Boltzmann constant. The chemical potential μ is determined by the constraint that the total number of photons in the system is N :

$$\sum_i \langle N_i \rangle = N. \quad (3)$$

The phenomenon of BEC for noninteracting photons is fully described by Eqs. (2) and (3). The nontrivial aspect is the determination of the chemical potential as a function of N and T . Once μ is known, all thermodynamic quantities like total energy, specific heat, and pressure follow directly from sums over the energy levels involving the occupation numbers in Eq. (2).

To this end, Eq. (2) can be rewritten as follows:

$$\langle N_i \rangle = \frac{ze^{-\beta E_i}}{1 - ze^{-\beta E_i}}, \quad (4)$$

where the fugacity z can be expressed as $z = \exp(\beta\mu)$ and $E_i = E_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}$. The ground state of the system is the state with $\mathbf{k} = \mathbf{0}$, and the energy of the ground state is zero. The BEC means that at low temperatures, a macroscopic number of photons occupies the ground state. From Eq. (4), the average number of photons in the ground state is $N_0 = 2z/(1 - z)$. The ground-state population diverges as $z \rightarrow 1$. When we split off the the ground-state population in Eq. (3), Eq. (3)

becomes

$$\frac{2z}{1-z} + \sum_{i \neq 0} \langle N_i \rangle = N. \quad (5)$$

At this point, we should note that the sum over i in Eq. (5) includes an integral over \mathbf{k} and a sum over σ . On putting Eq. (4) into Eq. (5), Eq. (5) is transformed into the following form:

$$\frac{2z}{1-z} + \frac{2V}{\pi^2(\beta\hbar c)^3} g_3(z) = N, \quad (6)$$

where $g_3(z)$ is a special case of the Bose functions $g_n(z)$. It is obvious that for real values of z between 0 and 1, $g_3(z)$ is a bounded, positive, monotonically increasing function of z . At $z = 0$, $g_3(0) = 0$, and at $z = 1$, $g_3(1) = 1.202$. To satisfy Eq. (6), it is necessary that $0 \leq z \leq 1$. The fugacity z can be determined numerically from Eq. (6). z is a function of temperature T , volume V , and photon number N . Once z is known, the average number of photons in the ground state can be obtained from the relation $N_0 = 2z/(1-z)$.

Let us rewrite Eq. (6) in the form,

$$N_0 + \frac{2V}{\pi^2(\beta\hbar c)^3} g_3(z) = N. \quad (7)$$

The critical temperature T_c can now be found by setting $N_0 = 0$ and $z = 1$ in Eq. (7). This results in the following expression for the critical temperature,

$$T_c = \frac{\hbar c}{k_B} \left[\frac{n\pi^2}{2g_3(1)} \right]^{1/3}, \quad (8)$$

where $n = N/V$ is the number density of photons. In the limit as $V \rightarrow \infty$, we obtain the solution of Eq. (6),

$$z = \begin{cases} 1, & T \leq T_c, \\ \text{the root of } g_3(z) = (T_c/T)^3 g_3(1), & T > T_c. \end{cases} \quad (9)$$

By virtue of Eq. (9), from Eq. (7) we find that the condensate fraction of photons in the ground state is given by

$$\frac{N_0}{N} = \begin{cases} 1 - \left(\frac{T}{T_c} \right)^3, & T \leq T_c, \\ 0, & T > T_c. \end{cases} \quad (10)$$

This means that a macroscopic number of photons occupies the ground state. This phenomenon is

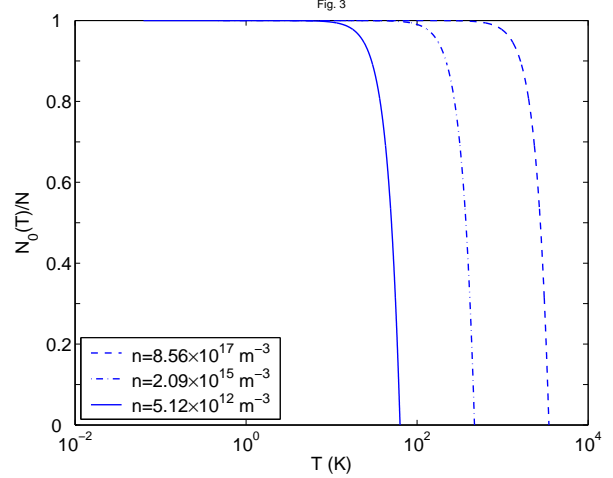


Fig. 2.— In an ideal white body, variation of the condensate fraction N_0/N with the temperature T and the number density n .

known as the Bose-Einstein condensation. The photons in the ground state form a condensate, which is superfluid.

From the above description, we can see that the BEC of three-dimensional photons in a white body takes place in the momentum space. As we have known, last-scattering occurred at a temperature of $k_B T \sim 0.3$ eV. The critical temperature T_c of photonic BEC should meet the condition $k_B T_c \lesssim 0.3$ eV. In other words, this condition is $T_c \lesssim 3481.35$ K. From Eq. (8), T_c is directly proportional to $n^{1/3}$. At $n = 5.12 \times 10^{12}$, 2.09×10^{15} , $8.56 \times 10^{17} \text{ m}^{-3}$, $T_c = 63.20, 469.03, 3480.95$ K, respectively. According to Eq. (10), the variation with T and n of condensate fraction N_0/N is shown in Fig. 2. For a fixed n , the condensate fraction decreases continuously from unity to zero as the temperature increases from zero to the critical temperature T_c . For a fixed T , the condensate fraction is a monotonically increasing function of n .

3. Radiation properties of an ideal white body

The universe before last-scattering was filled with a large number of thermal free electrons. The electromagnetic field within the universe before last-scattering is in thermodynamical equilibrium but has no conserved photon number (Turner

1999). Such equilibrium is established via the continual scattering of photons by thermal free electrons. The electromagnetic field in thermal equilibrium is called thermal radiation and characterized by a definite temperature T . The universe after last-scattering can keep the photon number conserving and thus becomes a white body. The thermal radiation in a white body is also called white-body radiation. The photons in a white body are in a thermal radiation state, which is called a normal state. In order to characterize the thermal radiation state, we need to conceive a grand canonical ensemble of photons. Some identical systems of the ensemble may be in an eigenstate of the Hamiltonian \mathcal{H}_{em} given by Eq. (1), while the distribution of the ensemble over the eigenstates is described by the density operator of the thermal radiation state

$$\rho = \frac{\exp(-\mathcal{H}_{em}/k_B T)}{\text{Tr} \exp(-\mathcal{H}_{em}/k_B T)}. \quad (11)$$

The basis states used in the trace are the eigenstates of the Hamiltonian \mathcal{H}_{em} . The main thermodynamic quantity in white-body radiation is the total energy E or the energy density $u = E/V$, which is the ensemble average of the corresponding microscopic quantity,

$$E = \sum_{\mathbf{k}\sigma} \hbar\omega_{\mathbf{k}} \langle N_{\mathbf{k}\sigma} \rangle. \quad (12)$$

Here we have utilized the average notation $\langle N_{\mathbf{k}\sigma} \rangle = \text{Tr}(\rho N_{\mathbf{k}\sigma})$.

It is easily found that the ensemble average of the number operator of photons in a mode $\mathbf{k}\sigma$ satisfies the well-known Bose-Einstein distribution given by Eq. (2). In the usual way altering the summation to an integration, we obtain

$$E = V \int_0^\infty \rho(\omega, T) d\omega, \quad (13)$$

$$\rho(\omega, T) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{z^{-1} e^{\hbar\omega/k_B T} - 1}, \quad (14)$$

where $z = \exp(\mu/k_B T)$ is the fugacity. Equation (14) gives the spectral energy density of white body radiation.

The spectral energy density $\rho(\omega, T)$ of white body radiation is not an observable. An observable of white body radiation is the spectral intensity $I(\omega, T)$ of white body radiation. $I(\omega, T)$

is defined as the power per unit surface area per unit solid angle per unit frequency emitted at a frequency ω by a white body. $I(\omega, T)$ is related to $\rho(\omega, T)$ through the relation:

$$I(\omega, T) = \frac{c}{4\pi} \rho(\omega, T) = \frac{\hbar}{4\pi^3 c^2} \frac{\omega^3}{z^{-1} e^{\hbar\omega/k_B T} - 1}. \quad (15)$$

In the condensation region of $T < T_c$, $z = 1$ and so $I(\omega, T)$ is identical with Planck's law for blackbody radiation. In the non-condensation region of $T > T_c$, $z < 1$ and so $I(\omega, T)$ becomes smaller than Planck's law for blackbody radiation at the same frequency and temperature. At $T = T_c$, $I(\omega, T)$ is continuous.

On putting Eq. (14) into Eq. (13), we obtain the following result:

$$E(T) = \frac{4\sigma}{c} V T^4 \frac{g_4(z)}{g_4(1)}, \quad (16)$$

where $\sigma = \pi^2 k_B^4 / 60 \hbar^3 c^2$ is called the Stefan-Boltzmann constant and $g_4(1) = \pi^4/90$. The energy density $u(T)$ of white body radiation is not an observable. Another observable of white body radiation is the total intensity $R(T)$ of white body radiation. $R(T)$ is defined as the total energy radiated per unit surface area of a white body per unit time. $R(T)$ is related to $u(T)$ through the relation:

$$R(T) = \frac{c}{4} u(T) = \sigma T^4 \frac{g_4(z)}{g_4(1)}. \quad (17)$$

In the condensation region of $T < T_c$, $z = 1$ and so $R(T)$ is identical with the Stefan-Boltzmann law for blackbody radiation. In the non-condensation region of $T > T_c$, $z < 1$ and so $R(T)$ becomes smaller than the Stefan-Boltzmann law for blackbody radiation. At $T = T_c$, $R(T)$ is continuous. It is interesting and important to note that all of the energy density arises from the nonzero momentum photons. That is, zero-momentum photons, however numerous, contribute nothing to $u(T)$.

For convenience, in Eq. (15) we set the number density of photons $n = 8.56 \times 10^{17} \text{ m}^{-3}$. The transition temperature T_c depends on the number density n . $T_c = 3480.95 \text{ K}$ at $n = 8.56 \times 10^{17} \text{ m}^{-3}$. Further, the fugacity z depends on the temperature T . We find that at $T = 2480.95, 4037.91, 7936.57, 17126.29 \text{ K}$, $z = 1.0, 0.7, 0.1, 0.01$, respectively. The spectral intensity of white body radiation at density $n = 8.56 \times 10^{17} \text{ m}^{-3}$ is plotted in

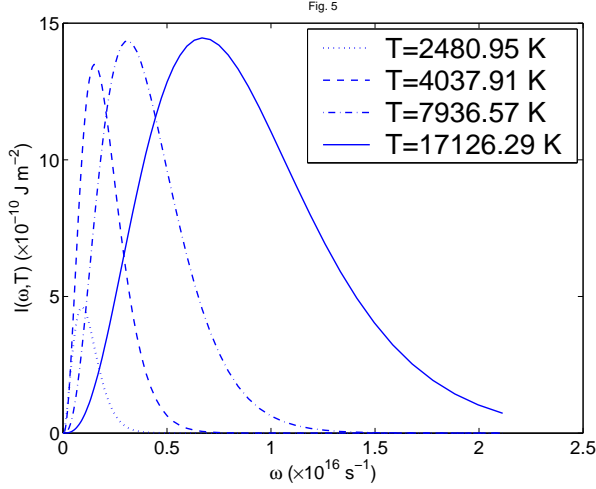


Fig. 3.— In an ideal white body, at a fixed density $n = 8.56 \times 10^{17} \text{ m}^{-3}$, variation of the spectral intensity $I(\omega, T)$ with the frequency ω and the temperature T .

Fig. 3 as a function of its frequency, where dotted, dashed, and dashed-dotted, and solid lines correspond to $T = 2480.95, 4037.91, 7936.57, 17126.29$ K, respectively. There are three features: (i) in the condensation region of $T < T_c$, $I(\omega, T)$ is identical with Planck's law for blackbody radiation; (ii) in the non-condensation region of $T > T_c$, $I(\omega, T)$ becomes larger and larger than Planck's law for blackbody radiation as temperature T ascends; (iii) the peak frequency of blackbody radiation at $T = 2480.95$ K is $\omega_m = 9.2 \times 10^{14} \text{ s}^{-1}$ while the peak frequency of white body radiation at $T > T_c$ becomes larger and larger as temperature T ascends. Next let us turn to Eq. (17). The total intensity of white body radiation is plotted in Fig. 4 as a function of its temperature, where the dashed, dashed-dotted, and solid lines correspond to $n = 5.12 \times 10^{12}, 2.09 \times 10^{15}, 8.56 \times 10^{17} \text{ m}^{-3}$, respectively. The dotted line represents variation of the total intensity of blackbody radiation with the temperature. In the non-condensation region of $T > T_c$, $R(T)$ becomes smaller than the Stefan-Boltzmann law for blackbody radiation. At a fixed density $R(T)$ is a monotonically increasing function of T and at a fixed temperature $R(T)$ is also a monotonically increasing function of n .

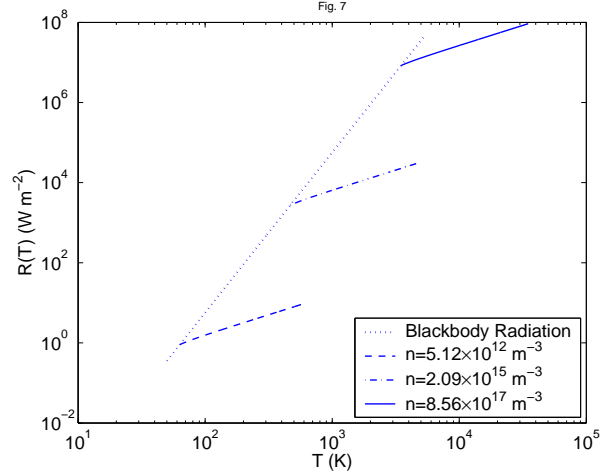


Fig. 4.— In an ideal white body, variation of the total intensity $R(T)$ with the temperature T and the density n . The dotted line represents variation of the total intensity $R(T)$ of blackbody radiation with the temperature T

4. Discussion

In investigating the BEC of three-dimensional noninteracting photons, we assume that the universe after last-scattering is an ideal white body that contains a tremendously large number of thermal photons ($n \lesssim 8.56 \times 10^{17} \text{ m}^{-3}$) and is at an extremely high temperature ($T \lesssim 3481.35$ K). This BEC theory is highly relevant to the present universe. The vacuum state of the universe before last-scattering is a true vacuum state of photons, in which $a_{\mathbf{k}\sigma}|0\rangle = 0$ for all \mathbf{k} . As the temperature of the universe descends, the universe after last-scattering undergoes a second-order phase transition into a BEC state. Consequently, the white-body radiation in the universe becomes the black-body radiation. The vacuum state of the present universe is a condensate or superfluid of photons. This vacuum state is defined as follows:

$$|N_0\rangle = \frac{1}{\sqrt{N_0!}} (a_{\mathbf{0}\pm}^\dagger)^{N_0} |0\rangle, \quad (18)$$

where N_0 is given by Eq. (10). This vacuum state satisfies the property: $a_{\mathbf{k}\sigma}|N_0\rangle = 0$ for all $\mathbf{k} \neq \mathbf{0}$. The observational findings in astronomy (Riess 1998; Perlmutter 1999) can be correctly interpreted in terms of a BEC theory of noninteracting thermal photons. Such a theory can account

for fluctuations of the vacuum and for the cosmic microwave background radiation (Penzias 1965).

The photons in the state $|N_0\rangle$ are virtual photons, which can not be detected by any means. Because of fluctuations of the vacuum, virtual photons becomes real photons with extremely short lifetime. Therefore, the vacuum state possesses the vacuum energy E_0 , as given by

$$E_0 = \sum_{\mathbf{k}\sigma}' \frac{1}{2} \hbar \omega_{\mathbf{k}}, \quad (19)$$

where the prime on the summation symbol means that $|\mathbf{k}| \leq k_m$. k_m is the maximum wave number of virtual photons and must satisfy the relation $k_m \ll m_e c / \hbar$, where m_e is the rest mass of electron. In the usual way altering the summation to an integration, we obtain $E_0 = V u_0$, where u_0 is the energy density of the vacuum and is given by $u_0 = \hbar c k_m^4 / 8\pi^2$. Although u_0 is very small, the volume V of the universe is enormous, so that E_0 is tremendously large. The vacuum energy E_0 is called the dark energy, because it can not be detected by any means. The dark energy perhaps drives the accelerating expansion of the universe (Riess 1998; Perlmutter 1999). In 1998, two research teams studied several dozen, distant type Ia supernovae and discovered that the universe is expanding at an ever-accelerating rate (Riess 1998; Perlmutter 1999). The discovery of cosmic acceleration is arguably one of the most important developments in modern cosmology. The physical origin of cosmic acceleration remains a deep mystery. A probable explanation of cosmic acceleration is that 75% of the energy density of the universe exists in a new form with large negative pressure, known as dark energy (Turner 1999; Huterer 1999). Dark energy is dark because its action is unknown. Two proposed forms for dark energy are the cosmological constant, a constant energy density filling space homogeneously (Peebles 2003; Weinberg 1989), scalar field models (Campo 2011) which comprehend, e.g., quintessence, phantom, K-essence, and tachyon fields.

Within the framework of quantum statistical mechanism, we investigate the BEC of three-dimensional photons in an ideal white body, which acts as a model of the universe after last-scattering. During the radiation-dominated stage of the development of the universe, the three-generation charged leptons are a unitary Dirac

field. In the meanwhile, the electromagnetic field and the Dirac field are in thermal equilibrium with each other. Under the surface of last scattering, the number of photons in the universe is conserved. The electromagnetic field in thermal equilibrium and with a conserved photon number is called a white-body radiation. After the temperature of the universe becomes smaller than 3481.35 K, the universe undergoes a second-order phase transition into a BEC state. Consequently, the white-body radiation in the universe becomes the blackbody radiation that we observe in the present universe. The vacuum that we currently observe is a condensate consisting of zero-momentum photons. This phase transition occurs in the matter-dominated stage of the development of the universe. The transition temperature of 3480.95 K is much higher than any temperature we normally encounter in the present universe.

In summary, we have proposed a BEC theory of three-dimensional photons in the universe after last-scattering, which can be regarded as an ideal white body. At zero temperature, we find that all photons in the universe condense into the zero-momentum state. The computation shows that the transition temperature T_c is about 3480.95 K. The photon system undergoes a second-order phase transition from the normal state to the BEC state. In the meanwhile, the white-body radiation in the universe becomes the blackbody radiation that we observe in the present universe. The predicted properties of the BEC of ideal thermal photons are highly relevant to the present universe.

This work was supported by the National Natural Science Foundation of China under Grants No. 10174024 and No. 10474025.

REFERENCES

- Anderson, M. H., Ensher, J. R., Matthews, M. R., Wieman, C. E., & Cornell, E. A. 1995, *Science*, 269, 198
- Bardeen, J., Cooper, L. N., & Schrieffer, J. R. 1957, *Phys. Rev.*, 108, 1175
- Bolda, E. L., Chiao, R. Y., & Zurek, W. H. 2001, *Phys. Rev. Lett.*, 86, 416
- Bradley, C. C., Sackett, C. A., Tollett, J. J., & Hulet, R. G. 1995, *Phys. Rev. Lett.*, 75, 1687

- Butov, L. V., Lai, C. W., Ivanov, A. L., Gossard, A. C., & Chemla, D. S. 2002, *Nature*(London), 417, 47
- del Campo, S., Fabris, J. C., Herrera, R., & Zimdahl, W. 2011, *Phys. Rev. D*, 83, 123006
- Chiao R. Y., & Boyce, J. 1999, *Phys. Rev. A*, 60, 4114
- Chiao, R. Y. 2000, *Opt. Commun.*, 179, 157
- Cooper, L. N. 1956, *Phys. Rev.*, 104, 1189
- Davis, K. B., Mewes, M. O., Andrews, M. R., van Druten, N. J., Durfee, D. S., Kurn, D. M., & Ketterle, W. 1995, *Phys. Rev. Lett.*, 75, 3969
- Demokritov, S. O., Demidov, V. E., Dzyapko, O., Melkov, G. A., Serga, A. A., Hillebrands, B., & Slavin, A. N. 2006, *Nature*(London), 443, 430
- Deng, H., Haug, H., & Yamamoto, Y. 2010, *Rev. Mod. Phys.*, 82, 1489
- Eisenstein J. P., & Macdonald, A. H. 2004, *Nature*(London), 432, 691
- Huterer, D., & Turner, M. S. 1999, *Phys. Rev. D*, 60, 081301
- Kirton P., & Keeling, J. 2013, *Phys. Rev. Lett.*, 111, 100404
- Klaers, J., Vewinger, F., & Weitz, M. 2010a, *Nature Phys.*, 6, 512
- Klaers, J., Schmitt, J., Vewinger, F., & Weitz, M. 2010b, *Nature*(London), 468, 545
- Klaers, J., Schmitt, J., Damm, T., Vewinger, F., & Weitz, M. 2012, *Phys. Rev. Lett.*, 108, 160403
- Nikuni, T., Oshikawa, M., Oosawa, A., & Tanaka, H. 2000, *Phys. Rev. Lett.*, 84, 5868
- Peebles, P. J. E., & Ratra, B. 2003, *Rev. Mod. Phys.*, 75, 559, and references therein
- Penzias A. A., & Wilson, R. W. 1965, *Astrophys. J.*, 142, 419
- Perlmutter, S. et al. (Supernova Cosmology Project), 1999, *Astrophys. J.*, 517, 565
- Planck, M. 1914, *The Theory of Heat Radiation*, 2nd edition, transl. M. Masius, (Philadelphia, PA: P. Blakiston's Son & Co.), p. 9
- Riess, A. G. et al. (High-Z Supernova Search), 1998, *Astron. J.*, 116, 1009
- Turner, M. S., & Tyson, J. A. 1999, *Rev. Mod. Phys.*, 71, S145, and references therein
- Weinberg, S. 1989, *Rev. Mod. Phys.*, 61, 1
- Zel'dovich Y. B., & Levich, E. V. 1969, *Soviet Phys. JETP*, 28, 1287